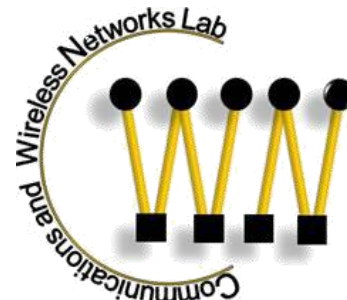


# Missing Spectrum-Data Recovery in Cognitive Radio Networks Using Piecewise Constant Nonnegative Matrix Factorization

Alireza Zaeemzadeh, Mohsen Joneidi, Behzad Shahrabi, and Nazanin Rahnavard  
School of Electrical Engineering and Computer Science  
University of Central Florida

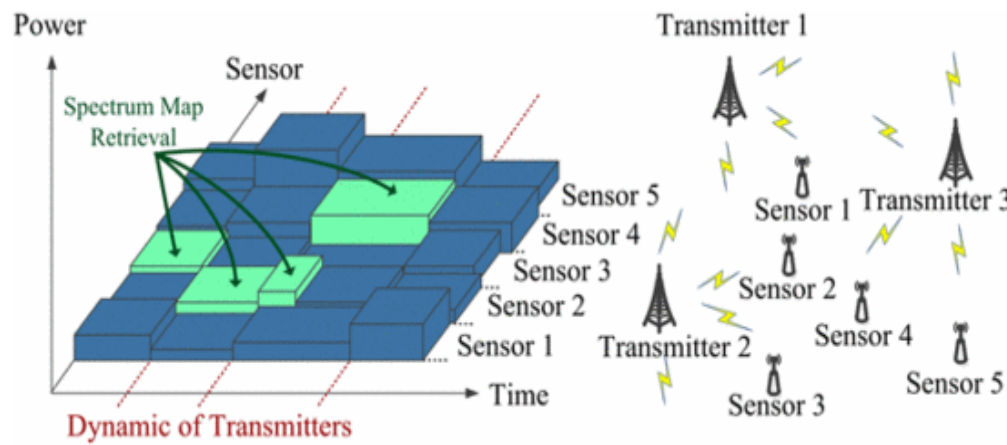


## Outline

- **Spectrum Sensing in Cognitive Radio Networks:** (which is prone to **missing data**)
- **Piecewise Constant Nonnegative Matrix Factorization**
- **Majorization-Minimization to Solve the PC-NMF Problem**
- **Results and Conclusion**

## Spectrum Sensing in Cognitive Radio Networks

- The radio frequency (RF) spectrum is a precious resource that must be utilized efficiently.
- The main feature of CRs is the opportunistic usage of spectrum
- CR systems try to improve the spectrum efficiency by using the spectrum holes in frequency, time, and space domains.



Picture source: Yu, Chung-Kai, and Kwang-Cheng Chen. "Spectrum map retrieval using cognitive radio network tomography." *GLOBECOM Workshops (GC Wkshps), 2011 IEEE*. IEEE, 2011.

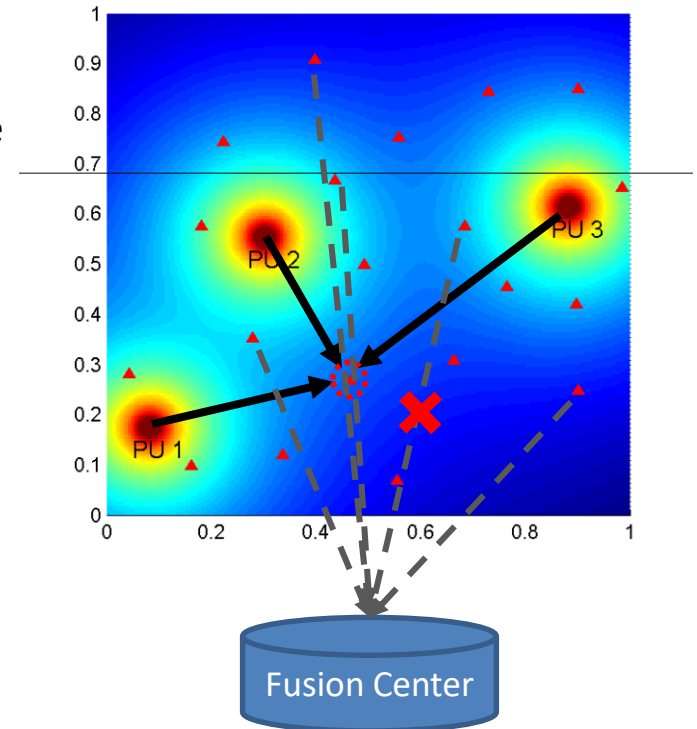
# Spectrum Sensing in Cognitive Radio Networks

- **PUs:** Primary Users or licensed users that have higher priorities to use the spectrum.
- **SUs:** Secondary Users or unlicensed users that try to utilize the spectrum in an opportunistic manner.

There are some **missing entries** among our data set.

- Hardware Limitations
- Energy Limitations
- Network Traffic
- Connection Failure

$$s_i(t) = \sum_{j=1}^{N_{PU}} p_j(t) \gamma_{ij}(t) + z_i(t)$$



# Piecewise Constant Nonnegative Matrix Factorization

The received power at  $i^{\text{th}}$  SU at time  $t$  can be formulated as:

$$s_i(t) = \sum_{j=1}^{N_{PU}} p_j(t) \gamma_{ij}(t) + z_i(t)$$

Vector Form:

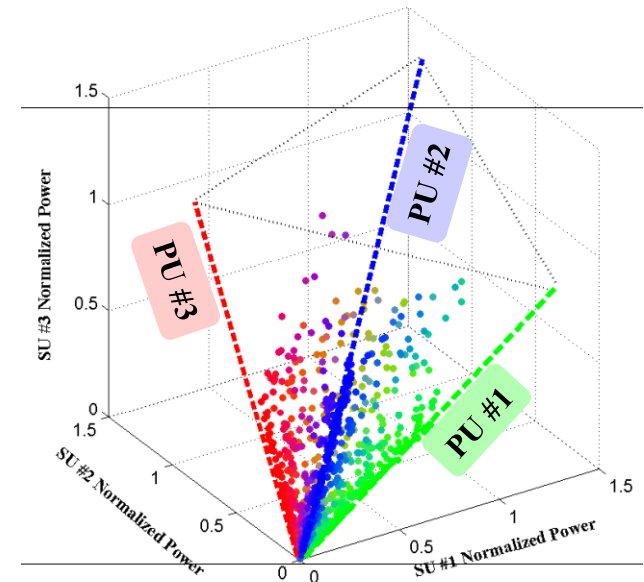
$$\mathbf{s}(t) = \sum_{j=1}^{N_{PU}} p_j(t) \boldsymbol{\gamma}_j(t) + \mathbf{z}(t)$$

Matrix Form:

$$\mathbf{S} = \mathbf{\Gamma} \mathbf{P} + \mathbf{Z}$$

$N_{PU} \times T$

$N_{SU} \times N_{PU}$



# Piecewise Constant Nonnegative Matrix Factorization

$$P_{\text{transition}} \ll 1$$

The power levels of PUs tend to be piecewise constant

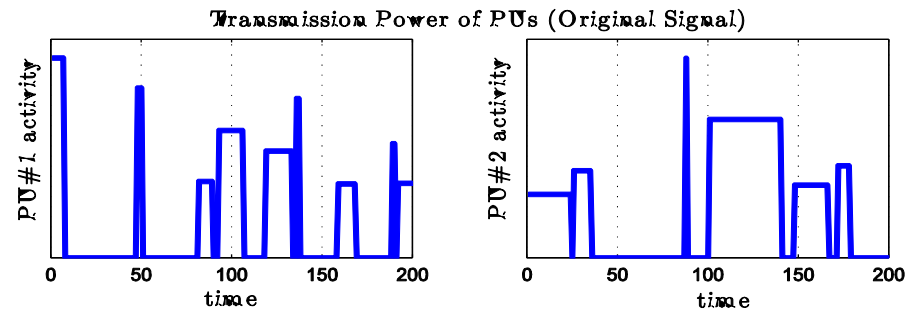
□  $D_W$  is a weighted measure of fit

□  $F(\mathbf{P})$  is a penalty which favors piecewise constant solutions

$$F(\mathbf{P}) = \sum_{t=2}^T \|p_t - p_{(t-1)}\|_0$$

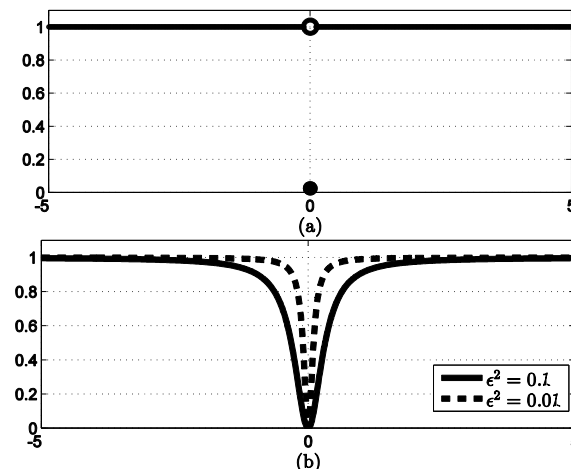
$$\text{minimize}_{\Gamma, \mathbf{P}} D_W(\mathbf{S}|\Gamma\mathbf{P}) + \beta F_\epsilon(\mathbf{P}),$$

$$\text{subject to } \Gamma \geq 0, \mathbf{P} \geq 0.$$



$$\text{minimize}_{\Gamma, \mathbf{P}} D_W(\mathbf{S}|\Gamma\mathbf{P}) + \beta F(\mathbf{P}),$$

$$\text{subject to } \Gamma > 0, \mathbf{P} > 0,$$



## Majorization-Minimization to Solve the PC-NMF Problem

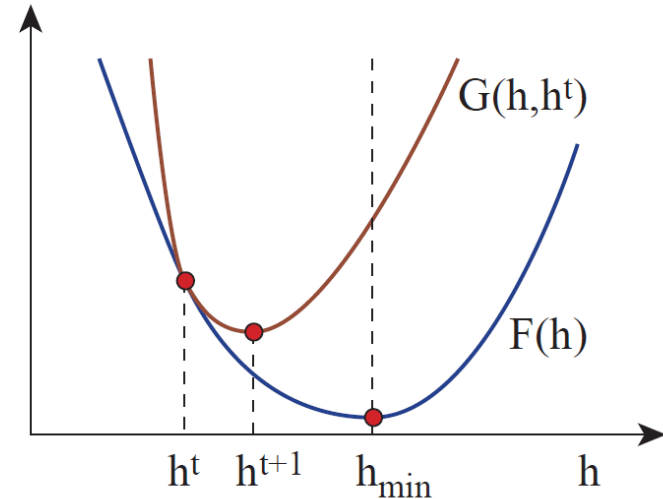
**Definition 1**  $G(h, h')$  is an auxiliary function for  $F(h)$  if the conditions

$$G(h, h') \geq F(h), \quad G(h, h) = F(h)$$

are satisfied.

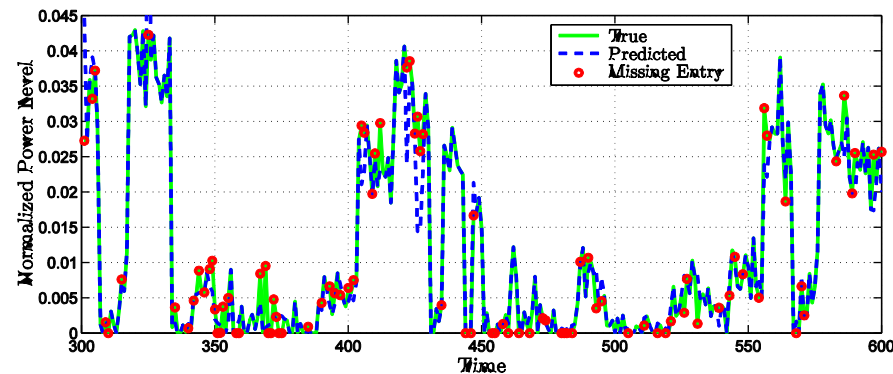
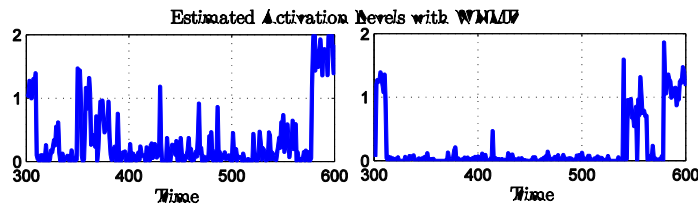
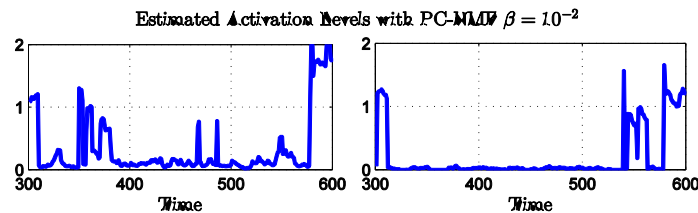
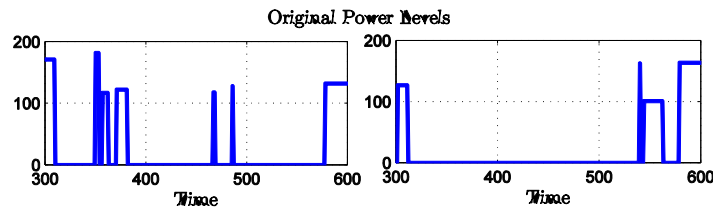
$$h^{t+1} = \arg \min_h G(h, h^t)$$

$$F(h^{t+1}) \leq G(h^{t+1}, h^t) \leq G(h^t, h^t) = F(h^t)$$





## Results



- The proposed method decreases the effect of noise and fading.
- The sharp transitions are preserved.



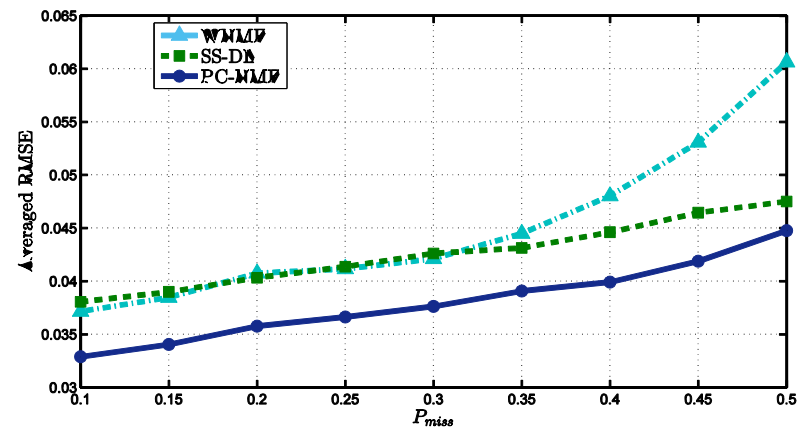
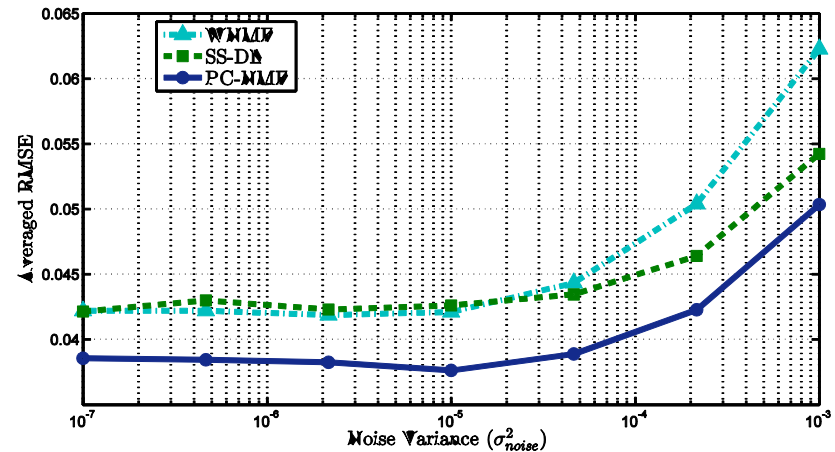
## Results

Performance of the PC-NMF vs Weighted and NMF and a method based on Dictionary Learning.

- About 10% less RMSE

MM methods converge much faster than methods based on gradient descent.

Method	Average Running Time (s)
SS-DL	10.3039
WNMF	0.0944
PC-NMF	0.0952



**Any Questions?**  
**Thank You**