Feedback Acquisition and Reconstruction of Spectrum-Sparse Signals by Predictive Level Comparisons

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***Abstract*—In this letter, we propose a sparsity promoting feedback acquisition and reconstruction scheme for sensing, encoding and subsequent reconstruction of spectrally sparse signals. In the proposed scheme, the spectral components are estimated utilizing a sparsity-promoting, sliding-window algorithm in a feedback loop. Utilizing the estimated spectral components, a level signal is predicted and sign measurements of the prediction error are acquired. The sparsity promoting algorithm can then estimate the spectral components iteratively from the sign measurements. Unlike many batch-based compressive sensing algorithms, our proposed algorithm gradually estimates and follows slow changes in the sparse components utilizing a sliding-window technique. We also consider the scenario in which possible flipping errors in the sign bits propagate along iterations (due to the feedback loop) during reconstruction. We propose an iterative error correction algorithm to cope with this error propagation phenomenon considering a binary-sparse occurrence model on the error sequence. Simulation results show effective performance of the proposed scheme in comparison with the literature.**

***Index Terms*—1-Bit compressive sensing (CS), binary-sparse error correction, level comparison (LC) sign measurements, sparse signal acquisition,.**

I. Introduction

SPECTRUM sparse signals arise in many applications such as cognitive radio networks, frequency hopping communications, radar/sonar imaging systems, musical audio signals and many more. In such cases, the signal components maybe sparsely spread over a wide spectrum and need to be acquired without prior knowledge of their frequencies. In this research, we propose a scheme and the corresponding signal processing algorithms for acquisition of spectrally sparse signals. The proposed scheme utilizes tools from the general theory of compressive sensing (CS) [1]–[4] to address spectral sparsity.

Several schemes have already been proposed for sparse signal acquisition. These include the random demodulator [5], the multicoset sampler [6], and the modulated wideband converter [7]. However, the acquired measurements need to be quantized and encoded to bits for subsequent transmission or processing. This is addressed in the quantized compressive sensing [8]–[11] literature.

The extreme case of 1-bit compressive sensing has been extensively studied [12]–[18] and proved to be robust against high levels of additive noise on the measurements [11]. However, the 1-bit measurements acquired in these works provide no information on the norm of the sparse signal. Hence in these works, reconstruction is possible only up to a scale factor.

In the proposed scheme, the input signal is compared with a level signal [19]–[22] and sign measurements of the error are acquired. The level signal is estimated adaptively in a feedback loop utilizing a sparse reconstruction algorithm. The reconstruction algorithm utilizes the previously acquired sign values to estimate the sparse signal components and predict the level signal, subsequently. This overcomes the scale ambiguity of 1-bit CS reconstruction.

The idea of acquiring sign measurements of level comparisons was also applied in [23]–[25]. Previous studies on one-bit sigma-delta quantization [26]–[28] investigate how adaptivity in the level values can improve the reconstruction error bound in terms of the number of measurements. The approach in [29] achieves exponential decay in the reconstruction error as a function of the number of measurements but requires the levels themselves to be transmitted for reconstruction. This is in contrast to our proposed scheme where the adaptive levels are estimated from the sequence of previously acquired sign measurements themselves. In Section IV, we provide performance comparisons with state-of-the-art techniques in [28], [29] and show effective performance of the proposed scheme by simulations.

In case the acquired sign bits are transmitted over a channel, the available sign bits to the receiver may contain flipping errors. Due to the feedback, these errors may propagate and make reconstruction unstable. To cope with this, we propose an iterative algorithm for correcting possible sign flip errors assuming a binary-sparse occurrence model on the error sequence. Unlike the previously proposed error-robust 1-bit CS reconstruction techniques [30]–[32], our proposed error correction algorithm

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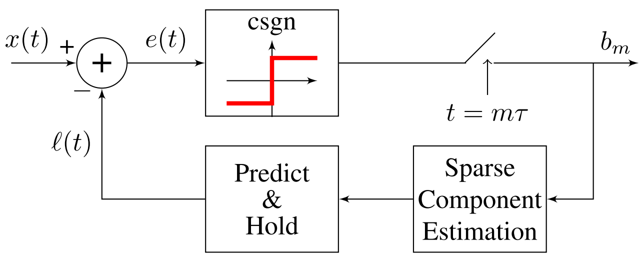

Fig. 1. Block diagram for the proposed acquisition scheme.

alleviates the need for prior knowledge of the number of errors by applying a binary-sparse occurrence model on the error sequence.

This letter is organized as follows. In Section II, we describe our proposed feedback acquisition and the corresponding reconstruction scheme. Section III presents the algorithms performed in the main building blocks of our proposed scheme. Section IV provides the simulation results and finally Section V concludes the paper.

For reproduction of the results reported in this letter, MATLAB files are posted on the personal webpage of M. Boloursaz Mashhadi at ee.sharif.edu/∼boloursaz.

II. The Proposed Acquisition and Reconstruction Scheme

We adopt the sparse exponential model in order to accommodate the general class of spectrally sparse signals that arise in many applications. Assuming that the power spectrum of the input signal $x(t)$ is sparse, $x(t)$ can be approximated by a summation of a few complex exponentials from the set $\{s_{z_i}(t)\}_{i=1}^{N}$ in which each exponential component can be predicted by $s_{z_i}(t+\epsilon) = e^{z_i \epsilon} s_{z_i}(t)$. In other words, assume that $x(t) = \sum_{z \in Z} s_z(t), Z = \{z_1, z_2, \cdots, z_N\}, z_i \in \mathbb{C}$ and $x(t)$ is sparse in the sense that only for a few number of z_is, the corresponding $s_{z_i}(t)$ exists in $x(t)$ and the other components are zero. Note that the adopted model allows nonequidistant frequencies and hybrid real/imaginary exponentials.

Fig. 1 shows the block diagram of the proposed feedback acquisition scheme. In this figure, the complex input signal $x(t)$ is compared with the level signal $\ell(t)$ utilizing a simple comparator. The error signal $e(t)$ goes through the complex sign block and is then sampled uniformly at $t = m\tau$ resulting in the output sequence of sign values $b_m \in \{\pm 1 \pm 1j\}$, where we define $\mathrm{csgn}(.) = \mathrm{sgn}(\mathrm{Re}(.)) + j\mathrm{sgn}(\mathrm{Im}(.))$, $\mathrm{sgn}(x) = 1$ for $x \geq 0$, $\mathrm{sgn}(x) = -1$ for $x < 0$, and $j = \sqrt{-1}$. The complex sign function $\mathrm{csgn}(.)$ operates elementwise on vectors. To encode the signal more efficiently, $\ell(t)$ is calculated from b_m in a feedback loop utilizing a sparse component estimation algorithm followed by prediction.

In many cases, the acquired signal needs to be subsequently transmitted over a channel. In these cases, the sign bits available for reconstruction at the receiver experience flipping errors. These errors cause the receiver to estimate inaccurate level values. If the levels estimated at the receiver are inaccurate, the subsequent sign bits received will be wrongly interpreted, which introduce further errors to reconstruction. In other words, due to the feedback, the error propagates and may destabilize the whole reconstruction. To prevent error propagation, we propose an iterative error correction scheme assuming a binary sparse occurrence model on the error sequence. At the receiver, single iterations of the component estimation and error correction algorithms are alternately applied to correct the sign-flip errors along reconstruction as depicted in Fig. 2.

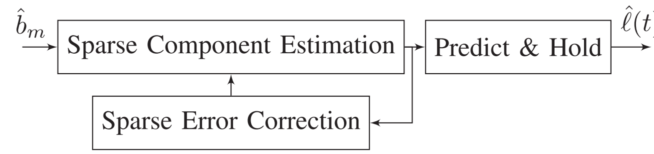

Fig. 2. Block diagram for reconstruction at the receiver.

In the next section, we elaborate the algorithms performed in the main building blocks of the proposed scheme.

III. The Proposed Algorithms

In this section, we first elaborate our proposed algorithm to be performed in the sparse component estimation block to reconstruct the spectral components from the sign bits. Then, we introduce our proposed sparsity-promoting algorithm to correct sign-flip errors at the receiver.

A. Sparse Component Estimation

Consider a sliding window on the input samples as $X_m = [x(m\tau), x((m-1)\tau), \ldots, x((m-M+1)\tau)]^T$ in which τ is the sampling period. Moreover, denote the corresponding level and sign values by $L_m = [\ell(m\tau), \ell((m-1)\tau), \ldots \ell((m-M+1)\tau)]^T$ and $B_m = [b_m, b_{m-1}, \ldots, b_{m-M+1}]^T$, respectively. Utilizing this vector notation, we get $B_m = \mathrm{csgn}(X_m - L_m)$. Now define $S_m = [s_{z_1}(m\tau), s_{z_2}(m\tau), \ldots, s_{z_N}(m\tau)]^T$ as the state vector for the observed signal $x(t)$, we can write $X_m = \Phi S_m$, where Φ is a Vandermonde matrix defined by

$$\Phi = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-z_1\tau} & e^{-z_2\tau} & \cdots & e^{-z_N\tau} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-z_1(M-1)\tau} & e^{-z_2(M-1)\tau} & \cdots & e^{-z_N(M-1)\tau} \end{bmatrix}. \quad (1)$$

The exponential modeling $s_{z_i}(t+\epsilon) = e^{z_i\epsilon} s_{z_i}(t)$ simplifies to a one step predictor as $S_m = P \odot S_{m-1}$ where $P = [e^{z_1\tau}, e^{z_2\tau}, \ldots, e^{z_N\tau}]$ and $\odot$ is element wise multiplication of two vectors. To estimate and update the sparse state vector S_m, we propose to iteratively minimize

$$\hat{S}_m = \arg\min_{S} \|\hat{B}_m - \mathrm{csgn}(\Phi S - L_m)\|_2^2 + \lambda_1 \|S - P\hat{S}_{m-1}\|_2^2 + \lambda_2 \sum_{i=1}^{N} g_\sigma([S]_i) \quad (2)$$

where $\hat{S}_m$ and $\hat{S}_{m-1}$ represent estimates of the vector of sparse components for the sliding windows corresponding to $t = m\tau$ and $t = (m-1)\tau$, respectively, and $[S_m]_i$ denotes the ith element of the vector S_m. Note that $\hat{B}_m$ is the vector of observed sign bits and is different from the true B_m in the sense that it may contain bit-flip errors. The first term of the cost function in (2) enforces consistency with the encoded sequence of sign values, the second term guarantees smooth update of the solution and the last term promotes sparsity.

For the sparsity promoting term, we set $g_\sigma(s) = \frac{\arctan(\sigma|s|)}{\arctan(\sigma)}$. It is easy to show that $\lim_{\sigma\to\infty} \sum_i g_\sigma([S]_i) = \|S\|_0$ and $\lim_{\sigma\to 0} \sum_i g_\sigma([S]_i) = \|S\|_1$. Thus, by starting from a small

σ value and increasing it along the iterations, we migrate from the convex ℓ_1 to the nonconvex ℓ_0 norm gradually. Note that the utilized arctan sparsity promoting term benefits from several advantages over alternatives utilized in the literature as discussed in [33]–[35]. Similarly, for ease of calculating the gradient, we replace the sign function with an S-shaped, infinitely differentiable function [36]–[38]. We set $f_\delta(s) = \frac{2}{\pi}\arctan(\delta s)$, for some $\delta > 0$ which is differentiable with the derivative $f'(s) = \frac{d}{ds}f(s) = \frac{2}{\pi}\frac{\delta}{1+\delta^2 s^2}$. It is obvious that $\lim_{\delta\to\infty} f_\delta(s) = \mathrm{sgn}\,(s)$ and hence we increase δ exponentially along the iterations. Making these substitutions we get (3)

$$\begin{aligned}\hat{S}_m &= \arg\min_S \ C(S)\\ &= \arg\min_S \|\hat{B}_m - \mathrm{c}f(\Phi S - L_m)\|_2^2\\ &\quad + \lambda_1\|S - P\hat{S}_{m-1}\|_2^2 + \beta\sum_{i=1}^{N}\arctan(\sigma|[S]_i|)\end{aligned} \tag{3}$$

where $\beta = \frac{\lambda_2}{\arctan(\sigma)}$ and $\mathrm{c}f(.) = f(\mathrm{Re}(.)) + jf(\mathrm{Im}(.))$ for any function $f : \mathbb{R} \mapsto \mathbb{R}$,. To solve (3), we shall find the roots of $\frac{\partial}{\partial S}C(S) = 0$. To reduce the computational cost, we apply only one iteration on each sliding window but gradually increase the σ and δ parameters along temporal iterations. Utilizing a sliding-window approach also enables following possible changes in the spectral components along the iterations. We have

$$\begin{aligned}&2\Phi^H \mathrm{c}f'(\Phi S - L_m) \odot (\mathrm{c}f(\Phi S - L_m) - \hat{B}_m)\\ &\quad + 2\lambda_1(S - P\hat{S}_{m-1}) + \beta G \odot S = 0\end{aligned} \tag{4}$$

where

$$[G]_i = \frac{1}{|[S]_i|(1 + \sigma^2|[S]_i|^2)}, \quad \text{for } i = 1, \cdots, N. \tag{5}$$

To solve this nonlinear equation, we approximate the first term in (4) by its value at the prior state estimate. Denoting $Y_{m-1} = 2\lambda_1 P\hat{S}_{m-1} - 2\Phi^H cf'(\Phi\hat{S}_{m-1} - L_m) \odot (cf(\Phi\hat{S}_{m-1} - L_m) - \hat{B}_m)$, we have

$$(2\lambda_1 \mathbb{1}_N + \beta G) \odot S = Y_{m-1} \tag{6}$$

where $\mathbb{1}_N = [1, \cdots, 1] \in \mathbb{R}^N$. The elements of $2\lambda_1\mathbb{1} + \beta G$ are $2\lambda_1 + \frac{\beta}{|[S]_i|(1+\sigma^2|[S]_i|^2)}$, which are all real positive values, therefore, from (6) we obtain

$$\angle[S]_i = \angle[Y_{m-1}]_i, \tag{7}$$

$$2\lambda_1|[S]_i| + \frac{\beta}{1 + \sigma^2|[S]_i|^2} = \alpha_i \tag{8}$$

where $\alpha_i = |[Y_{m-1}]_i|$. We can rewrite (8) as the following cubic polynomial equation in terms of $r_i = |[S]_i|$

$$2\lambda_1\sigma^2 r_i^3 - \alpha_i\sigma^2 r_i^2 + 2\lambda_1 r_i + (\beta - \alpha_i) = 0. \tag{9}$$

The coefficients of the cubic polynomial (9) are real. Hence (9) has either three real roots or a single real root and a complex conjugate pair. To enforce sparsity, coefficients with smaller amplitudes are encouraged and hence (3) is minimized by choosing the smallest nonnegative real root of (9). We propose to solve (9) as follows.

Case 1: All roots of (9) are real: The sum of the three roots $\frac{\alpha_i\sigma^2}{2\lambda_1\sigma^2} = \frac{\alpha_i}{2\lambda_1} > 0$ is always positive and hence there exists at least a positive root. The smallest positive root is feasible for the algorithm.

Case 2: One of the roots is real and two are complex conjugate pair of each other. The real root is positive and hence feasible if the product of the roots $\frac{\alpha_i-\beta}{2\lambda_1\sigma^2}$ is positive. Hence we enforce $\beta = \frac{\lambda_2}{\arctan(\sigma)} < \alpha_i$ or equivalently increase σ such that $\sigma > \tan\left(\frac{\lambda_2}{\alpha_i}\right)$. Note that as σ increases iteratively not only is this condition enforced but also the last term of (3) tends to ℓ_0-norm of S.

As described above, the magnitude and phase of $[\hat{S}_m]_i$ are given by the solution of (9) and (7), respectively.

Using the state estimate $\hat{S}_m$, the predict and hold block calculates the next level value as $\ell((m+1)\tau) = \sum_{i=1}^{N}[P \odot \hat{S}_m]_i$. Finally, to get $\ell(t)$ from its samples, each $\ell(m\tau)$ is held by this block at the output for as long as the sampling period τ.

B. Sparse Error Correction

Let us define the real and imaginary error vectors E_m^r and E_m^i with elements $e_m^r, e_m^i \in \{0,1\}$. Define $e_m^r = 1$ if $\mathrm{Re}(b_m)$ is flipped and $e_m^r = 0$, otherwise. Hence, we get $\mathrm{Re}(b_m) = \mathrm{Re}(\hat{b}_m)(1 - 2e_m^r)$ and $\mathrm{Im}(b_m) = \mathrm{Im}(\hat{b}_m)(1 - 2e_m^i)$. Note that for ease of calculations, we consider the real and imaginary error vectors separately and provide our algorithm for the real part. The imaginary part is similar. It is obvious that E_m^r is a sparse vector with elements in $\{0,1\}$. Hence, we propose an iterative scheme to update and estimate E_m^r by solving (10). At the receiver, single iterations of optimizations (10) and (2) are applied alternately to correct the flipping errors while estimating the sparse spectral components, simultaneously. Let us denote $\hat{\hat{E}}_{m-1}^r = \mathcal{S}(\hat{E}_{m-1}^r)$, in which the $\mathcal{S}(\cdot)$ operator denotes sliding the estimated error vector for one sample and inserting a zero as the initial estimate of its new element. Now to estimate E_m^r, we solve the following:

$$\begin{aligned}\hat{E}_m^r &= \arg\min_E h(E) + \theta\sum_{i=1}^{M}[E]_i\\ \text{s.t.}\quad &\|E - \hat{\hat{E}}_{m-1}^r\|_2 \le \epsilon, \quad [E]_i \in \{0,1\}\end{aligned} \tag{10}$$

where the second term of the cost function is the ℓ_1 norm which promotes sparsity in E_m^r since the elements of E_m^r are nonnegative.

Now let us relax the range constraint for $[E_m^r]_i$ to the convex interval $[0,1]$. Also note that $h(E) = \|\mathrm{Re}(\hat{B}_m) \odot (1 - 2E) - \mathrm{sgn}\,(\mathrm{Re}(\Phi\hat{S}_m - L_m))\|_2^2$ is a quadratic convex term with regard to E. We solve the resulting convex optimization by projected gradient steps followed by stochastic rounding [39], [40] to $\{0,1\}$. Note that both the projected gradient and stochastic rounding techniques have convergence guarantees for the convex case. To perform this, each iteration consists of two steps. The first step updates T_m in the reverse gradient direction of (10) by

$$T_m = \hat{\hat{E}}_{m-1}^r - \epsilon\frac{D}{\|D\|_2} \tag{11}$$

TABLE I
MSEs (dB) ACHIEVED BY OUR PROPOSED SCHEME

		$k = 2.5\%$	$k = 5\%$	$k = 10\%$	$k = 20\%$
$p = 0$		-19.9	-19.6	-17.9	-10.4
$p = 0.0125$	w/o EC	-16.3	-12.1	-10.3	-6.3
	w/ EC	-19.4	-18.3	-14.5	-7.8
$p = 0.025$	w/o EC	-10.2	-8.7	-4.6	F
	w/ EC	-18.2	-17.4	-12.5	-7.1
$p = 0.05$	w/o EC	-4.1	-2.3	F	F
	w/ EC	-17.8	-16.5	-10.2	-5.8

where ϵ is an small step-size and

$$D = -4\mathrm{Re}(\hat{B}_m) \odot (\mathrm{Re}(\hat{B}_m) \odot (1 - 2\hat{\hat{E}}^r_{m-1}) - \mathrm{sgn}\,(\Phi\hat{S}_m - L_m)) + \theta\mathbb{1}_M. \quad (12)$$

The second step simultaneously performs projection and stochastic rounding by

$$[\hat{E}_m]_i = \begin{cases} 0, & [T_m]_i \leq \mathbf{u} \\ 1, & [T_m]_i > \mathbf{u} \end{cases} \quad (13)$$

where $\mathbf{u}$ is generated as a uniformly distributed random variable over the interval $[0, 1]$.

IV. Simulation Results

To numerically evaluate the performance of our proposed scheme, we generate random spectrally sparse signals according to the model presented in Section II with $N = 500$, $M = 50$, $\tau = 5 \times 10^{-4}$ sec, and $Z = \{1j, 2j, \ldots, 500j\} \times \omega_0$, $\omega_0 = 10$ rad/sec. The nonzero spectral components are selected uniformly at random and the corresponding amplitudes come from a $\mathcal{N}(0, 1)$ distribution. For comparisons, we average the normalized reconstruction mean square error (MSE $= \frac{\|S - \hat{S}\|_2^2}{\|S\|_2^2}$) values over 100 runs after 25 iterations of the algorithm. Table I reports the resulting NMSE for different sparsity factors. Simulation results confirm that once the algorithm parameters are optimized for a special simulation scenario (sparse component distribution and variance, sparsity factor, window length, prequantization noise variance, error rate, etc.), the same set of parameters work sufficiently well for all simulation instances in that scenario. Hence, for the results reported in Section IV, we experimentally optimized the parameters as $\lambda_1 = 0.1$, $\lambda_2 = 0.05$, $\delta_0 = 1$, $\sigma_0 = 10$, $\theta = 0.1$, $\epsilon = 0.05$, $\delta_m = 1.01 \times \delta_{m-1}$, and $\sigma_m = 1.1 \times \sigma_{m-1}$.

In Table I, p denotes the rate at which sign-flip errors occur, the sparsity factor k is defined as the ratio of the number of nonzero spectral components over the total number of components N, "w/ EC" and "w/o EC" represent the results with and without the proposed error correction (EC) iterations and the letter "F" shows divergence of the proposed algorithm (MSE>-5 dB) due to the error propagation phenomenon. As shown, EC is necessary to avoid error propagation.

Next, we investigate the general scenario in which $x(t)$ both contains frequencies that do not lie on any of the quantized frequencies $Z = \{1j, 2j, \ldots, 500j\} \times 10$ rad/sec (the off-grid problem) and may also have stable real exponential parts. Note that $\exp\,(\gamma t + j(K\omega_0 + \Delta\omega)t) = \exp\,((\gamma + j\Delta\omega)t)\exp\,(jK\omega_0 t), \gamma \leq 0$ that is the grid frequency $\exp\,(jK\omega_0 t)$ with an amplitude that varies with time according to $\exp\,((\gamma + j\Delta\omega)t)$. Hence, if γ and $\Delta\omega$ are small, the algorithm will still be able to converge and follow the smooth changes in the component amplitudes. To investigate this, we generate $x(t)$ with a sparsity factor of $k = 0.05$ that contains components on $\omega = j214.8 \times 10, -1.5 + j442.1 \times 10$ rad/sec and provide the MSE curves versus iteration in Fig. 3. These curves confirm effective performance of the proposed algorithm to follow smooth changes in the component amplitudes.

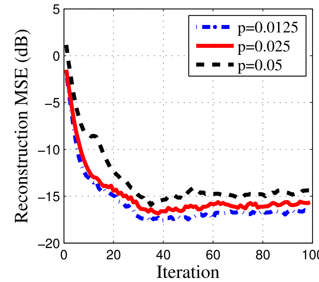

Fig. 3. MSE vs. Iteration for the Off-Grid Scenario ($k = 0.05$).

TABLE II
MSE COMPARISONS (dB) WITH THE LITERATURE

	$M = 50$	$M = 100$	$M = 200$
[23]	-13.34	-18.54	-25.88
[24]	-15.76	-29.61	-57.21
Proposed (ℓ_1 Regularized)	-14.54	-28.96	-56.17
Proposed (Arctan Regularized)	-16.12	-29.73	-56.96

In Table II, we compare the performance of our proposed algorithm with state-of-the-art techniques in [28], [29] for different values of the window length M where $k = 5\%$, $p = 0$ and the other simulation parameters are fixed as previously. This table provides the final normalized reconstruction MSEs (dB) achieved by the three acquisition and reconstruction schemes averaged over 20 runs when there exists an additive zero-mean Gaussian prequantization noise with standard deviation 0.1. As observed in this table, both our proposed algorithm and [29] outperform [28] especially for larger values of M. This is due to an exponential error decay bound for our proposed algorithm and [29] in comparison with a root exponential decay bound for the $\Sigma\Delta$ scheme in [28]. We also investigate the improvement achieved by utilizing the arctan sparsity promoting function over the popular ℓ_1 regularization in this table. To this end, we consider $g([S]_i) = \|[S]_i\|_1$ and utilize a subgradient based soft thresholding algorithm to solve optimization (2). The results show that arctan improves the performance especially for smaller values of the window length M.

V. Conclusion

In this letter, we proposed a feedback acquisition scheme for encoding of spectrally sparse signals to a stream of 1-bit measurements. We proposed a sparsity promoting reconstruction algorithm to predict comparison levels in a feedback loop to facilitate more efficient 1-bit measurements of the input signal. We also proposed a sparse error correction technique to cope with possible sign flip errors during transmission.

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