New Results on Unequal Error Protection Using LDPC Codes

Nazanin Rahnavard and Faramarz Fekri

***Abstract*— In this letter, we propose a new scheme to construct low-density parity-check (LDPC) codes that are suitable for unequal error protection (UEP). We derive UEP density evolution (UDE) formulas for the proposed ensemble over the binary erasure channel (BEC). Using the UDE formulas, high performance UEP codes can be found. Simulation results depict an improvement in the bit error rate of more important bits in comparison with previous results on UEP-LDPC codes.**

***Index Terms*— Unequal error protection, low-density parity-check codes, density evolution.**

I. Introduction

Unequal Error Protection (UEP) property is very desirable for applications where different bits have different significance. The first UEP codes were proposed by Masnick and Wolf [1]. Later, other UEP design methodologies were developed, e.g., [2]. Because of the outstanding performance of LDPC codes, it is desirable to have unequal error protection using LDPC codes. In [3], authors proposed a scheme for UEP-LDPC codes. The method in [3] is based on the conventional bipartite Tanner graph. Here, we propose another scheme based on combining two Tanner graphs. We derive density evolution formulas for this ensemble over the BEC channel. Simulation results show that we can achieve improved performance by the proposed method.

The paper is organized as follows. In Section II, a new scheme for designing UEP codes is given. Section III describes an efficient encoding scheme for a special class of the proposed codes. Finally, we conclude the paper in Section IV.

II. A New Scheme for UEP

Suppose we want to transmit k information bits with two levels of importance over a BEC with erasure probability ϵ. To do this, we want to design an (n, k) UEP-LDPC code $\mathcal{C}$ having rate $R = k/n$. Let $k_M = \alpha k$ $(0 < \alpha < 1)$ be the number of more important bits (MIB) and $k_L = (1-\alpha)k$ be the number of less important bits (LIB). Let $m = n - k$ be the number of parity bits (PB). Let us define $G(n, m)$ as the Tanner graph corresponding to $\mathcal{C}$ with n variable nodes and m check nodes. Let H denote an $m \times n$ binary parity-check matrix corresponding to $G(n, m)$. We assume that H is full rank.

Before explaining our design criteria, it is good to provide some insight as to how an LDPC code can have different error protection levels for different bits. It is known that it is best to have high degrees for variable nodes. This is because the more information a variable node receives from its adjacent check nodes, the more accurately it can judge about its correct value. The previous scheme on UEP-LDPC codes [3] is based on having different degrees for MIB, LIB, and PB. Fig. 1 shows the ensemble proposed in [3], where the authors derived density evolution for the UEP case. It was concluded that MIB have larger degrees than LIB and there is a large gap among the BERs of MIB and LIB.

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The authors are with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA (e-mail: {nazanin, fekri}@ece.gatech.edu).

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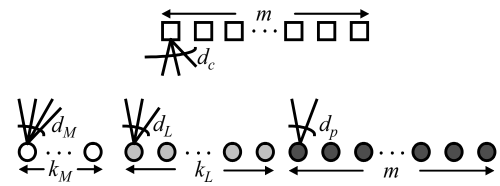

Fig. 1. The Tanner graph of the ensemble in [3] for the UEP property.

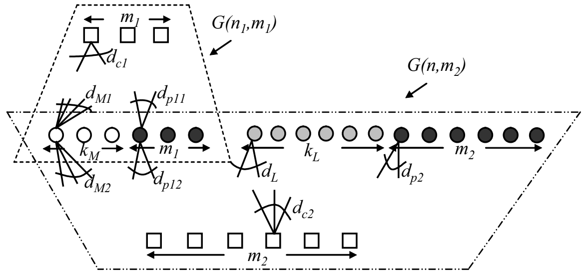

Fig. 2. The Tanner graph of the proposed ensemble.

To further reduce the error rates for the MIB, we propose another scheme. In this scheme, we combine two Tanner graphs. The first Tanner graph corresponds to a high-rate LDPC code that is for protecting MIB. The second graph is for protecting all the data. The first Tanner graph has the role of determining the values of those bits in MIB that the second graph failed to determine. Therefore, the error rate for MIB can be reduced. Let $G_1 = G(n_1, m_1)$ and $G_2 = G(n, m_2)$ denote the first and second graph, respectively. Here $m_1 = \gamma m$ and $m_2 = (1-\gamma)m$ for some $0 < \gamma < 1$. The proposed ensemble is depicted in Fig. 2. Let us call the proposed ensemble as G_c. The first n_1 variable nodes in G_c, are protected by both G_1 and G_2. It should be noted that not all of these n_1 bits can be taken as information bits. In the following lemma we show that we have $n_1 - m_1$ information bits in this part of the codeword.

Lemma 1: Consider two Tanner graphs $G_1 = G(n_1, m_1)$ and $G_2 = G(n, m_2)$ that are combined to form an ensemble as in Fig. 2. Then, n_1 variable nodes that are common in both graphs contain $n_1 - m_1$ information bits.

Proof: Let H_1 and $H_2 = [H_{21}|H_{22}]$ denote $m_1 \times n_1$ and $m_2 \times n$ parity-check matrices corresponding to G_1 and G_2, respectively. It is easy to see that the parity-check matrix of the combined code is given by

$$H = \begin{pmatrix} H_1 & 0 \\ H_{21} & H_{22} \end{pmatrix}.$$

Since H is full rank, we have H_1 and H_{22} are also full rank. Since H_{22} is full rank, we conclude that all the first n_1 bits can potentially be information bits (their values can be set independently). However, since H_1 is also full rank with rank m_1, we conclude that only $n_1 - m_1$ bits of the first n_1 bits are information bits and the values of the other m_1 bits are determined by the values of the $n_1 - m_1$ information bits. This completes the proof. ■

We consider all of these $n_1 - m_1$ bits as MIB, i.e., $k_M = n_1 - m_1$. To impose different protection levels for MIB and LIB, it is necessary to know the positions of MIB and LIB in G_c. The following lemma states necessary and sufficient conditions for arranging MIB, LIB, and parity bits as in Fig. 2. The corresponding codeword is in the form of $\underline{c} = [MIB|P_1|LIB|P_2]$, where the parity bits have been divided into two parts P_1 and P_2.

Lemma 2: Let $H_1 = [A|H_{p1}]$ and $H_2 = [B|C|E|H_{p2}]$ denote the parity-check matrices that correspond to G_1 and G_2, respectively. Here A, H_{p1}, B, C, E, and H_{p2} are matrices of size $m_1 \times k_M$, $m_1 \times m_1$, $m_2 \times k_M$, $m_2 \times m_1$, $m_2 \times k_L$, and $m_2 \times m_2$, respectively. The assumption of separating MIB, LIB, and PB as shown in Fig. 2 is valid if and only if H_{p1} and H_{p2} are full rank.

Proof: Let us define H_p as

$$H_p = \begin{pmatrix} H_{p_1} & 0 \\ C & H_{p_2} \end{pmatrix}.$$

The columns of H_p correspond to the PB if and only if H_p is full rank. This is possible if and only if H_{p_1} and H_{p_2} are full rank. ■

Next we derive density evolution formulas for the proposed ensemble.

A. UEP Density Evolution (UDE)

Here, we derive the UDE formulas for the proposed ensemble. See Fig. 2 for the definitions of d_{M1}, d_{M2}, d_{p11}, d_{p12}, d_L, d_{p2}, d_{c1}, and d_{c2}. Let $M_{1,i}$ and $p_{11,i}$ denote the expected fractions of erasure messages that are received by the check nodes in G_1 from the variable nodes that correspond to MIB and P_1, respectively. Let $M_{2,i}$, $p_{12,i}$, L_i, and $p_{2,i}$ denote the expected fractions of erasure messages that are received by the check nodes in G_2 from the variable nodes that correspond to MIB, P_1, LIB, and P_2, respectively. Let M_i and $p_{1,i}$ denote the expected fractions of erasure messages that are sent to an incident edge from the variable nodes that correspond to MIB and P_1, respectively. Let also q_i (r_i) denote the probability that an erasure message is passed from the check nodes to the variable nodes in G_1 (G_2). Note that subscript i is the iteration number. The UDE formulas for $i \geq 0$ are given by

$$M_{1,0} = M_{2,0} = L_0 = p_{11,0} = p_{12,0} = \epsilon,$$

$$M_{1,i+1} = \epsilon q_i^{d_{M1}-1} r_i^{d_{M2}}, \quad M_{2,i+1} = \epsilon r_i^{d_{M2}-1} q_i^{d_{M1}},$$
$$p_{11,i+1} = \epsilon q_i^{d_{p11}-1} r_i^{d_{p12}}, \quad p_{12,i+1} = \epsilon r_i^{d_{p12}-1} q_i^{d_{p11}},$$
$$L_{i+1} = \epsilon r_i^{d_L-1}, \quad p_{2,i+1} = \epsilon r_i^{d_{p2}-1},$$
$$M_{i+1} = \frac{d_{M1}M_{1,i+1} + d_{M2}M_{2,i+1}}{d_{M1} + d_{M2}},$$
$$p_{1,i+1} = \frac{d_{p11}p_{11,i+1} + d_{p12}p_{12,i+1}}{d_{p11} + d_{p12}},$$
$$q_i = 1 - (1 - \lambda_{d_1} M_{1,i} - \lambda_{d_2} p_{11,i})^{d_{c1}-1},$$
$$r_i = 1 - (1 - \lambda_{d_3} M_{2,i} - \lambda_{d_4} p_{12,i} - \lambda_{d_5} L_i - \lambda_{d_6} p_{2,i})^{d_{c2}-1},$$

where λ_{d_1}, λ_{d_2} are the fractions of edges that are connected to the MIB and P_1 in G_1, respectively. Furthermore, λ_{d_3}, λ_{d_4}, λ_{d_5}, and λ_{d_6} are the fraction of edges that are connected to the MIB, P_1, LIB, and P_2 in G_2, respectively. These parameters are obtained by $\lambda_{d_1} = \frac{\alpha R d_{M1}}{T_1}$, $\lambda_{d_2} = \frac{\gamma(1-R)d_{p11}}{T_1}$, $\lambda_{d_3} = \frac{\alpha R d_{M2}}{T_2}$, $\lambda_{d_4} = \frac{\gamma(1-R)d_{p12}}{T_2}$, $\lambda_{d_5} = \frac{(1-\alpha)Rd_L}{T_2}$, and $\lambda_{d_6} = \frac{(1-\gamma)(1-R)d_{p2}}{T_2}$, in which $T_1 = \alpha R d_{M1} + \gamma(1-R)d_{p11}$ and $T_2 = \alpha R d_{M2} + \gamma(1-R)d_{p12} + (1-\alpha)Rd_L + (1-\gamma)(1-R)d_{p2}$.

Using the UDE formulas, the asymptotic behavior of a code with a given degree distribution can be estimated. Moreover, we can optimize the degrees so that we have low error rates for MIB while keeping the overall performance comparable to other codes. For a given R and α, optimal values for d_{M1}, d_{M2}, d_{p11}, d_{p12}, d_L, d_{p2}, d_{c1}, d_{c2}, and γ need to be found. However, we have two equality constraints imposed by edge constraints. These constraints are given by

$$\alpha R d_{M1} = \gamma(1-R)(d_{c1} - d_{p11}),$$
$$\alpha R d_{M2} + \gamma(1-R)d_{p12} + (1-\alpha)Rd_L = (1-\gamma)(1-R)(d_{c2} - d_{p2}).$$

Therefore, we require to optimize seven independent variables. We considered d_{c2} and γ as dependent variables. By setting some upper bounds for the degrees, we can search through all the possible values for degrees and select the ones that result in very low error rates for MIB. The cost function is considered as M_I (for some large integer I for which M_I is very close to its steady state value). It should be noted that the rate of the code corresponding to G_1 is given by $R_p = \frac{\alpha R}{\alpha R + \gamma(1-R)}$. For a fixed R and α, the larger is γ, the smaller are R_p and BER for MIB. On the other hand, we need to keep R_p large such that the performance of LIB remains acceptable. Therefore, we impose a lower bound on the rate R_p. Note that since the UDE formulas represent the asymptotic performance, every code obtained by the UDE formulas would not be necessarily optimal for finite-length codes. Therefore, we further refine the solutions for finite-length codes by choosing the one that has highest performance using iterative decoding.

B. Simulation Results

Consider the problem of designing a rate 1/2 UEP code with $\alpha = 0.1$. Let us assume the following search space: $d_{M1}, d_{M2}, d_L \leq 25$, $d_{p11}, d_{p12}, d_{p2} \leq 5$, $d_{c1}, d_{c2} \leq 15$, and $R_p \geq 0.8$. Using the UDE formulas, we optimize the codes. For example, we picked a code that results in $M_I = 0$ and $L_I = 7.9 \times 10^{-31}$ for $\epsilon = 0.45$ and $I = 1000$ iterations. This code also results in $M_I = 2.85 \times 10^{-26}$ and $L_I = 2.49 \times 10^{-12}$ for $\epsilon = 0.45225$. Table I summarizes the degrees for the optimized code.

For the finite-length case, we found the BERs versus the channel erasure probability for this code when the code length is $n = 4000$ ($k_M = 200$, $k_L = 1800$, $m_1 = 28$, $m_2 = 1972$) and the maximum number of decoding iterations is 200. Fig. 3

TABLE I
DEGREE DISTRIBUTIONS OF THE PROPOSED RATE $1/2$ UEP-LDPC CODE.

d_{M1}	d_{M2}	d_{p11}	d_{p12}	d_L	d_{p2}	d_{c1}	d_{c2}	γ
1	22	2	2	3	2	9	7	0.0143

shows the performance of the proposed code compared with the previous code presented in [3] with $d_M = 23$, $d_L = 3$, $d_p = 2$, and $d_c = 7$. Fig. 3 shows that the performance of MIB has improved by about one order of magnitude. On the other hand, the performance of LIB has degraded slightly for large ϵ, although LIB does not show an error floor as opposed to the code in [3]. We also included the BERs for P_1 and P_2 in the figure. Although P_1 has a total degree which is much smaller than that of MIB, the BER performance of P_1 and MIB are close. This is because the only neighbors of the check nodes in G_1 are MIB and P_1. Hence, certain messages from MIB help P_1 to be determined.

We also illustrated the performance of the proposed code when $n = 1000$ ($k_M = 50$, $k_L = 450$, $m_1 = 7$, $m_2 = 493$) in Fig. 4. For comparison, we depicted the performance of the code presented in [3] and a BEC-optimized irregular code, referred to as Code 1, found from [4] by setting the maximum allowable degree to 25. The degree distribution for Code 1 is given by $\lambda(x) = 0.24976x + 0.24716x^2 + 0.148x^5 + 0.003326x^6 + 0.35174x^{19}$ and $\rho(x) = x^7$. We showed the performance of $k_M = 50$ highest degree nodes (as MIB) and rest of the nodes separately for Code 1. We note that the performance of MIB in the proposed code is by far (three orders of magnitude for $\epsilon = 0.38$) better than the performance of MIB in Code 1.

III. Efficient Encoding

Here we present an efficient encoding scheme for the case $d_{p11} = 2$ and $d_{p2} = 2$, which happens in many optimized cases. H_{p1} and H_{p2} are full rank matrices by Lemma 2. It can be seen easily that H_{p1} (H_{p2}) is either an $m_1 \times m_1$ ($m_2 \times m_2$) dual-diagonal matrix Q_1 (Q_2) or its column permutation. Let $H_{p1} = Q_1\Pi_1$ and $H_{p2} = Q_2\Pi_2$ for some random permutation matrices Π_1 and Π_2. A systematic generator matrix for the parity-check matrix

$$H = \begin{pmatrix} A & Q_1\Pi_1 & 0 & 0 \\ B & C & E & Q_2\Pi_2 \end{pmatrix},$$

is given by

$$G = \begin{pmatrix} I_{k_M \times k_M} & A^T Q_1^{-T}\Pi_1 & 0 & (B^T + A^T Q_1^{-T}\Pi_1 C^T) Q_2^{-T}\Pi_2 \\ 0 & 0 & I_{k_L \times k_L} & E^T Q_2^{-T}\Pi_2 \end{pmatrix}.$$

It can be easily verified that $GH^T = 0$. The matrix Q_1^{-T} (Q_2^{-T}) corresponds to a differential encoder whose transfer function is $\frac{1}{1 \bigoplus D}$ [5]. We assumed that the information bits are $\underline{I} = [MIB|LIB]$, and therefore codewords are in the form of $\underline{c} = [MIB|P_1|LIB|P_2]$.

IV. Conclusion

A new design method for high performance UEP-LDPC codes was investigated. We proposed an ensemble that is a combination of two conventional bipartite graphs to improve the performance of more important bits (MIB). We derived unequal density evolution (UDE) formulas over the BEC for this ensemble. Using the UDE formulas, we are able to optimize the codes. We compared our results with the previous results on UEP-LDPC code presented in [3] for two lengths $n = 1000$ and $n = 4000$. We noted that the proposed method significantly decreased the BERs for the MIB. Moreover, the performance of the code for less important bits improved for small channel erasure probabilities. For $n = 1000$, we also compared the proposed code with a BEC-optimized code by setting a subset of the highest degree nodes with the corresponding size as MIB. The results showed the superiority of our code. Finally, an efficient encoding scheme for a special case of the proposed method was developed.

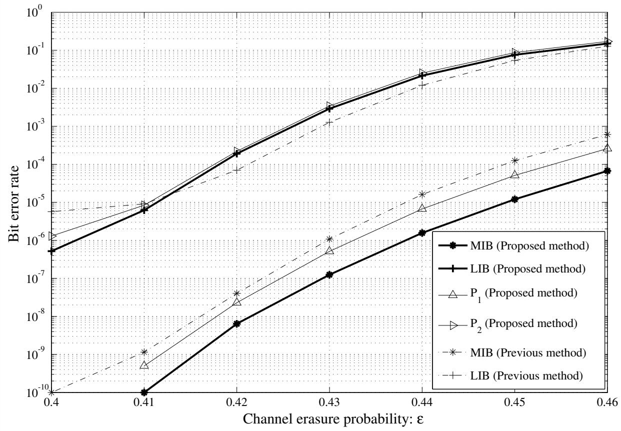

Fig. 3. Comparison between the proposed method and the method in [3]. The codes are of length $n = 4000$, rate $1/2$, and $\alpha = 0.1$.

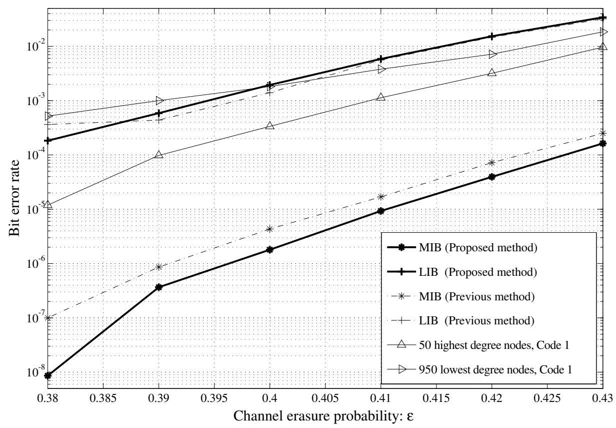

Fig. 4. Comparison between the proposed method, the method in [3], and Code 1. All the codes are of length $n = 1000$ and rate $1/2$.

References

[1] B. Masnick and J. Wolf, “On linear unequal error protection codes,” *IEEE Trans. Inform. Theory*, vol. 3, pp. 600-607, Oct. 1967.
[2] I. Boyarinov and G. Katsman, “Linear unequal error protection codes,” *IEEE Trans. Inform. Theory*, vol. 27, pp. 168-175, Mar. 1981.
[3] N. Rahnavard and F. Fekri, “Unequal error protection using low-density parity-check codes,” in *Proc. IEEE International Symposium on Inform. Theory*, p. 449, June/July 2004.
[4] http://lthcwww.epfl.ch/research/ldpcopt/
[5] M. Yang, Y. Li, and W. Ryan, “Design of efficiently encodable moderate-length high-rate irregular LDPC codes,” *IEEE Trans. Commun.*, vol. 52, pp. 564-571, Apr. 2004.